

Remark: The exam consists of four questions and all questions have to be answered. Total available time is 120 minutes. A dictionary as well as a calculator (fulfilling the requirements of the examination office) can be used.

Question 1 (30 points)

Please answer the following questions:

- a) What is the average and marginal tax rate? Give a verbal and formal explanation.
- b) What is the condition for a progressive, proportional, and regressive tax schedule, respectively? Answer each case verbally and formally.
- c) Show graphically the tax burden, average tax rate, and marginal tax rate as a function of the income for each of the following tax schedules. Decide whether the respective tax schedule is progressive, proportional or regressive. What is the main difference between both tax schedules?
 - i. Up to a certain income level, income is not subject to a tax. Every income above this income level is taxed at a constant tax rate.
 - ii. Up to a certain income level, income is not subject to a tax. If the income exceeds this income level, the whole income is taxed at a constant tax rate.
- d) Suppose a quantity tax is imposed on the seller's side in a competitive market. Show graphically the excess burden, tax revenue, and total welfare as a function of the tax rate.

Question 2 (30 points)

Consider a market with only one supplier. The inverse demand function is $p(x) = 13.5 - 2x$, where x is the quantity of the good and $0 \leq x \leq 6.75$. The cost function of the firm is $C(x) = 0.25x^2 + 15$. Please answer the following questions and round each result to two decimal places.

- a) What are the price and quantity as well as the consumer, producer, and total surplus in the market when the firm acts as a monopolist? What is the profit of the firm?
- b) Suppose that the firm is forced to set a price which is equal to the average cost. Calculate the price, quantity, consumer surplus, producer surplus, total surplus, and the firm's profit for this case.
- c) Suppose the firm has to set a price which is equal to the marginal cost. Calculate the price, quantity, consumer surplus, producer surplus, total surplus, and the firm's profit for this case.

- d) What problem occurs in part c)? Suppose the state has two alternatives to overcome this problem: lump sum subsidization and per unit subsidization. Which alternative is preferable and what total subsidy should then be paid?
- e) The solution in part b) is called second best solution and the solution in part c) is called first best solution. Give a short explanation for this differentiation.

Question 3 (30 points)

Consider two individuals, A and B, who have preferences over a pure public good q given by the demand functions, $q_A = 30 - p$ (for $p \leq 30$, 0 otherwise) and $q_B = 20 - p$ (for $p \leq 20$, 0 otherwise). The marginal cost of the public good is 20.

- a) Derive the optimal level of public good provision. What is the consumer surplus at this optimal level? What would be the consumer surplus if only individual A contributed? Briefly explain this result.
- b) The government uses Lindahl pricing to derive the level of public good provision and imposes a cost share of 0.6 on individual A and 0.4 on individual B. Why are the decisions of the two individuals not compatible at these cost shares? Briefly explain your answer.
- c) Derive the cost share for individual A and B at which the decisions of the two individuals are compatible. What is the level of public good provision at these cost shares?
- d) Now consider q is a private good and the two individuals have the same demand functions as above. Explain the difference in methods for finding the aggregate marginal willingness to pay curves for a private and a public good. What is the equilibrium quantity and price in the market for the private good if its marginal cost of production is 5?

Question 4 (30 points)

Consider a firm who produces a good x with price $p_x = 32$. The firm faces the cost function $C(x) = x^2$. Through the production of x the firm creates units of pollution a , which adversely affects a second firm in its production of a good y . The amount of pollution is proportional to the production of x . Specifically, we assume $a = x$. The cost function of the second firm is $C(y) = y^2 + ay$. The market price of y is $p_y = 40$.

- a) Determine the equilibrium quantities of x and y and the corresponding profits when the firms profit maximise separately.
- b) Determine the optimal quantities of x and y and the corresponding joint profit. Explain why the amount of x differs here to that in part a).
- c) What tax would have to be set per unit of x to reach an efficient outcome? What problem could a government face trying to implement this optimal tax rate?
- d) What redistributive measure would have to be taken such that no one is worse off compared to the initial situation? Explain briefly why the possibility of such redistribution may be of interest to policy setters interested in efficiency. (*Hint: Think about what constitutes an efficiency improvement and think about what the outcome after the tax would be if such redistribution actually took place.*)