Examination: 20052

International Marketing

Summer Semester 2011

Dr. John E. Brennan

You are allowed to use a non-programmable calculator (in accordance with the instructions given by the examination office) and a translating dictionary from your native language to English (without any notes written into it). **All** of the **seven** (7) examination questions must be answered. This examination consists of **three** (3) pages and must be completed within 60 minutes.

Question 1: NOTE: This problem MUST be solved using the table method presented in this lecture course. Other statistical methods for the solution of this problem will not be accepted.

A group of airline pilots undergo routine screening as part of a government mandated Drug Testing Program. The random variable Y= y, where y = 1 indicates the pilot tested uses an illegal substance and y = 0 the absence thereof. Within this group of pilots it is known that 0.2% of them are actual drug users. When the pilots are tested, only 92% of those who are actual drug users will test positive. The random variable X= x is the test result, x = 1 indicates a positive test and x = 0 a negative test result. The test is not perfect because only 96% of those pilots, who do not use any drugs at all, will receive a negative test result.

		X		
		1	0	$f_2(y)$
Y	1			
	0			
	$f_1(x)$			1.0

Conditional (X | Y= y)

		X		
		1	0	
Y	1			1.0
	0			1.0

Conditional (Y | X= x)

		X	
		1	0
Y	1		
	0		
		1.0	1.0

Bayesian Multiple Table

		X	
		1	0
Y	1		
	0		

a. What is the probability that a pilot who tests positive is not a drug user?

b. What is the probability that a pilot that has a negative test is actually a drug user?

Please turn to page 2

Question 2: Bayes' Theorem provides a logical framework for analyzing the human thought process and shows the usefulness of information in the assessment of future outcomes.

$$\Pr(Y \mid X = x) = \delta \Pr(Y), \text{ where } \delta = \Pr(X \mid Y = y) / \Pr(X).$$

a. Using Question 1 as an example: What are the prior probabilities of Y?

b. Explain in detail under what conditions the multiple δ is equal to one.

Question 3: Below is a joint probability distribution for the random pair (Y, X):

Y \ X	**1.5**	**3.5**	**5.5**
0.4	0.045	0.080	0.045
0.6	0.120	0.120	0.120
0.8	0.085	0.300	0.085

a. Compute E(Y).

b. Compute $E(Y \mid X = 1.5)$.

c. Compute C(X, Y).

d. Are X and Y stochastically independent?

Question 4: The two discrete random variables X= x and Y= y, have a known joint probability distribution, $(X, Y) \sim f(x, y)$, where $x = \{16, 17, 18\}$ and $y = \{0.5, 1.2, 3.7\}$.

a. Explain how the univariate marginal distributions are derived.

b. By normalizing the columns of the joint probability distribution the univariate conditional probability distributions of the random variables (Y|X=x) are derived. Are the conditional variances, V(Y|X=x), all equal to each other?

Question 5: When the range of a random variable, Y= y $\{y \geq a\}$ is restricted, we say that the random variable is truncated at point a.

a. Consider a discrete random variable, Y= y, where $y = \{2, 4, 6, 8\}$, with associated discrete uniform probabilities $\{1/4, 1/4, 1/4, 1/4\}$. If this random variable is truncated at $a = 4$ (therefore, $y > 4$), what are the truncated probabilities that $y = 2$ and that $y = 8$?

b. Compute the E(Y).

c. Compute the $E(Y \mid Y = y > 4)$.

Please turn to page 3

Question 6: Decision-makers make yes / no type decisions very often. This decision can be modeled as a dichotomous random variable, Y= y where y = {0, 1}. Furthermore, assume that this decision-maker has been provided with two highly relevant numerical sources of information, X = x and Z = z.

a. Assume that $E(Y \mid X=x, Z=z) = \alpha + \beta x + \gamma z$: What are the shortcomings of this model specification, if any?

b. The conditional random variable $(Y \mid X=x, Z=z)$ has a Bernoulli distribution, therefore, the conditional expectation, $E(Y \mid X=x, Z=z)$, provides what information?

Question 7: Consider a certain student is applying for admission at the O-v-G-Universität in Magdeburg (M= m, where m = 1 is accepted and m = 0 is rejected) and the Martin Luther Universität in Halle (H= h, where h = 1 is accepted and h = 0 is rejected). The student believes that there is a probability of 0.6 of being accepted in Halle and 0.2 of being accepted in Magdeburg. The probability of being rejected by both is 0.4.

		M		
		1	0	$f_2(h)$
H	1			
	0			
	$f_1(m)$			1.0

Conditional (M | H= h)

		M		
		1	0	
H	1			1.0
	0			1.0

Conditional (H | M= m)

		M	
		1	0
H	1		
	0		
		1.0	1.0

Bayesian Multiple Table

		M	
		1	0
H	1		
	0		

a. What is the probability that this student will be accepted in Halle if that student has already been accepted in Magdeburg?

b. Is the event "accepted in Magdeburg" independent from the event "accepted in Halle"? Explain your answer.

This is the End of the Examination
GOOD LUCK !