

Examiner: Prof. Dr. Peter Reichling

You are welcome to use non-programmable pocket calculators as well as English language dictionaries without any markings. This examination comprises 4 problems (on 3 pages). All of the problems are to be solved. Derivations of the formulas from the lecture or the exercise are not (!) required.

Good luck!

Examination Questions (Total Number of Points: 60)

Problem 1 (Downside Risk – 11 Points)

Consider two stocks, A and B , with normally distributed and perfectly positively correlated returns. Stock A shows an expected return of 10% p.a. and a volatility of 10%, while stock B has an expected return of 20% p.a. and a volatility of 30%.

- (a) Assuming that short sales of stocks are not (!) possible, plot possible portfolios of stocks A and B in a (μ, σ) -diagram. (2 points)
- (b) If possible, determine the shares x_A and x_B of stocks A and B , respectively, in an optimal portfolio according to
 - (i) ROY's criterion using a target return of 15% p.a.,
 - (ii) KATAOKA's criterion using a shortfall probability of 33% (use the table at page 3),
 - (iii) TELSER's criterion using a target return of 15% p.a. and a shortfall probability of 33%, or justify why such a portfolio does not exist. (9 points)

Problem 2 (Stochastic Dominance – 15 Points)

The random future return of stock A , \tilde{R}_A , has the following density function:

$$f_A(r) = \begin{cases} 10 & \text{if } 10\% \leq r < 20\%, \\ 0 & \text{otherwise.} \end{cases}$$

The density function of the random future return \tilde{R}_B of stock B is given as follows:

$$f_B(r) = \begin{cases} 20 & \text{if } 11\% \leq r < 14\%, \\ \frac{1}{4} & \text{if } 14\% \leq r < 30\%, \\ 0 & \text{otherwise.} \end{cases}$$

- (a) Plot the cumulative distribution functions $F_A(r)$ and $F_B(r)$ which correspond to the density functions $f_A(r)$ and $f_B(r)$, respectively, in the same diagram. (6 points)
- (b) Does stochastic dominance of
 - (i) first,
 - (ii) second
 order exist? If so, which cumulative distribution function dominates the other? Justify your answers. (5 points)

- (c) Assuming rationality of investors according to BERNOULLI's principle, do investors exist who prefer stock A to stock B and who are
- (i) "greedy" (strictly positive marginal utility),
 - (ii) risk-averse (strictly positive and strictly decreasing marginal utility)?
- Justify your answers. (4 points)

Problem 3 (Value at Risk of a Put Option – 17 Points)

There is a long position of an at-the-money put (!) option on the DAX with a strike price of 7,000.0 points and a maturity of 6 months in a bank's trading book. The DAX volatility is assumed to be 20.00%. The (discretely compounded) current 6-month Euribor equals 4.400% p.a.

- (a) Compute (in index points and according to the BLACK-SCHOLES-MERTON model)
 - (i) the value of the option (use the table at page 3),
 - (ii) the option's delta,
 - (iii) the option's gamma,
 - (iv) the option's volatility using the following assumptions: the DAX value is the only relevant risk factor; there is a quadratic relation between the change in the DAX value and the change in the value of the option; the change in the value of the DAX is normally distributed with a mean of zero.

(15 points)
- (b) Use your results of parts (a)(i) and (a)(iv) to compute the option's value at risk (in €) according to the current rules concerning internal models of the Basel Committee on Banking Supervision. (2 points)

Problem 4 (Discriminative Power of Rating Functions – 17 Points)

At time $t = 0$ rating agencies "Luck" and "Chance" carried out ratings of 100 companies, both using 3 rating classes but different rating functions. Observations at time $t = 1$ lead to the following contingency table of agency "Luck":

Rating at time $t = 0$	Observation at time $t = 1$	
	Default	Non-default
C	5	27
B	4	27
A	1	36

In case of agency "Chance", the following contingency table results:

Rating at time $t = 0$	Observation at time $t = 1$	
	Default	Non-default
c	7	9
b	1	36
a	2	45

- (a) Use one (and the same) diagram to show the receiver operating characteristic (ROC) curve of
- the rating function of agency "Luck",
 - the rating function of agency "Chance",
 - the perfect rating function,
 - the random rating function.

Compute (or just write down) and interpret all 4 (!) area under curve (AUC) values. (13 points)

- (b) Do the rating functions of agencies "Luck" and "Chance" exhibit discriminative power? Which rating function can differentiate better between companies of high and low creditworthiness? Justify your answers. (4 points)

**Distribution Function $N(x)$ of the Standard Normal Distribution
for Non-negative Arguments x**

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9983	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9990	0.9993	0.9995	0.9997	0.9998	0.9998	0.9999	0.9999	1.0000