

Examination: Reverse Logistics Management (1624)  
Semester: Winter Semester 2004/5  
Examiner: Prof. Dr. Karl Inderfurth

Permitted Aids: English dictionary (English into to any other language) without any handwritten entries !

Instructions: Complete exactly two (2) of the following three (3) questions. If all 3 are completed, only the first two will be evaluated. All questions will be weighted equally and are designed to be completed within 30 minutes.

### Question 1:

A smelter obtains amounts of each item (measured in mass units) and refines it, receiving revenue per mass unit of each metal. In order to process each metal, we need at least a concentration (percentage of mass)  $B$  mass units of it, and must pay the setup cost  $F$ . Each mass unit of items processed results in item-independent variable costs. The objective function is profit maximizing and contains the revenue generated from the metal recovery, cost for setting up to recover metals, and the per unit variable cost (independent of item type). Constraint (1) connects  $X$  and  $G$  decision variables, (2) ensures setups are paid for, (3) ensures maximum available amount of items at each recycler is not exceeded by decision variable, (4) ensures minimum mass concentration is met, if metal is recovered, (5) ensures that our capacity is not violated, and (6) that decision variables take on meaningful values.

$i = 1, \dots, I$  Item type index

$j = 1, \dots, J$  Recycler index

$k = 1, \dots, K$  Metal type index

$A_{i,k}$  Amount of metal  $k$  found per mass unit of item  $i$  (Mass)

$B_k$  Minimum concentration of metal  $k$  needed if refined (% of Mass)

$C$  Our capacity (in Mass)

$F_k$  Setup cost to recover metal  $k$

$G_{j,k}$  Amount of metal  $k$  processed from recycler  $j$  (Mass) (decision variable!)

$M$  A very large number

$R_k$  Revenue generated per (mass) unit of metal  $k$

$T_k$  Efficiency at processing metal  $k$  (percentage)

$V$  Processing cost per (mass) unit

$W_{i,j}$  Weight of item  $i$  available for processing at recycler  $j$  (Mass)

$X_{i,j}$  Amount (in mass) of item  $i$  processed from recycler  $j$  (decision variable!)

$Y_{j,k}$  Binary: Setup for metal  $k$  arriving from recycler  $j$  (decision variable!)

- a) Using the above given notation, put forth the objective function and constraints of the model.
- b) If our capacity were given not in mass units *for all items* as above, but rather in mass units *for each item*, how would the model above change? Appropriately define the capacity parameter and write which changes in the model become necessary.

## Question 2:

A firm engaged in remanufacturing requires parts gained from the disassembly of returned products as an input for the remanufacturing process. A certain demand for each part (or leaf) is given over a multi-period planning horizon. The firm faces a so-called *disassemble-to-order* problem, where parts can be gained either through disassembly or external procurement. Objective function will be cost minimizing and the constraints will (1) keep track of inventory for all items (leaves, cores and intermediate items), and (2) ensure decision variables take on meaningful values. Note in this problem, we assume that disassembly and procurement occur instantaneously (no lead times), that we can obtain any amount of cores from the market, and that maximum inventory levels are not specified. Note also that starting inventory values would be given as  $y_{i,0}, y_{j,0}, y_{k,0}$  for cores, intermediate items, and leaves respectively.

- a) Use the following notation to derive the objective function and constraints:

$i \in R$      Root items

$j \in I$      Intermediate items

$k \in L$      Leaf items

$t = 1, \dots, T$      Time periods ( $T$  = Planning horizon)

$c_i^p$  : Purchase cost of root  $i$     $i \in R$

$c_k^p$  : Purchase cost of leaf  $k$     $k \in L$

$c_i^s$  : Separation cost of root  $i$     $i \in R$

$c_j^s$  : Separation cost of intermediate  $j$     $j \in I$

$c_i^h$  : Holding cost of root  $i$     $i \in R$

$c_j^h$  : Holding cost of intermediate  $j$     $j \in I$

$c_k^h$  : Holding cost of leaf  $k$     $k \in L$

$c_j^d$  : Disposal cost of intermediate  $j$     $j \in I$

$c_k^d$  : Disposal cost of leaf  $k$     $k \in L$

$D_{k,t}$  : Demand for leaf  $k$  at time  $t$     $k \in L$     $t = 1, \dots, T$

$\pi_{i,j}$  : Amount of intermediate  $j$  obtained from disassembly of root  $i$     $i \in R$     $j \in I$

$\pi_{j,k}$  : Amount of leaf  $k$  obtained from disassembly of intermediate  $j$     $j \in I$     $k \in L$

- $x_{i,t}^p$  : Procurement of root  $i$  at time  $t$   $i \in R$   $t = 1, \dots, T$
- $x_{k,t}^p$  : Procurement of leaf  $k$  at time  $t$   $k \in L$   $t = 1, \dots, T$
- $x_{i,t}^s$  : Separation of root  $i$  at time  $t$   $i \in R$   $t = 1, \dots, T$
- $x_{j,t}^s$  : Separation of intermediate  $j$  at time  $t$   $j \in I$   $t = 1, \dots, T$
- $x_{j,t}^d$  : Disposal of intermediate  $j$  at time  $t$   $j \in I$   $t = 1, \dots, T$
- $x_{k,t}^d$  : Disposal of leaf  $k$  at time  $t$   $k \in L$   $t = 1, \dots, T$
- $y_{i,t}$  : Inventory of root  $i$  at end of period  $t$   $i \in R$   $t = 0, \dots, T$
- $y_{j,t}$  : Inventory of intermediate  $j$  at end of period  $t$   $j \in I$   $t = 0, \dots, T$
- $y_{k,t}$  : Inventory of leaf  $k$  at end of period  $t$   $k \in L$   $t = 0, \dots, T$

- b) For a problem with 2 cores, 3 intermediate items, 4 leaves, and 3 time periods in the planning horizon, write out the inventory balance constraints for the *intermediate* items in the *second* period ( $t = 2$ ).
- c) Discuss how the model would have to be changed in order to eliminate the modelling of intermediate items (i.e. a complete disassembly assumption, where intermediate items are immediately disassembled further to the leaves).

### Question 3:

Describe in detail the *Product Process Matrix* of Hayes and Wheelwright and its extension for remanufacturing environments by Guide et al. (2003).