Examination: 20023 – Option Pricing Theory   **Winter Term 2012/2013**

Examiner: Prof. Dr. Peter Reichling

Time available: 60 minutes

Aids permitted: non-programmable pocket calculators;
English dictionaries without any markings.

The examination comprises **five** problems. **FOUR** of them are to be solved. Please, choose one of the problems in Section 4! Intermediate results are to be rounded to four decimal places while end results to two decimal places. Answers to all problems must be given in **English**. Good luck!

Examination Questions (60 Points Total):

Problem 1 (Trading Strategies – 16 Points)

Suppose that you are long a covered call position and short a protective put position. Assume an identical strike for both options. (Hint: A protective put is long the underlying and long a put. A covered call is long the underlying and short a call.)

a) Express the *gross* payoff profile of this strategy both formally and graphically. What other option strategy does your portfolio remind you of? Would you pay or receive net premiums on this position? What is the implicit view taken on the volatility of the stock price and its direction if you hold this position? (12 points)

b) Suppose that the options on the Volkswagen shares (strike price €25) are trading at €1.25 (calls) and €0.30 (puts). State the stock price range at maturity in which the *net* payoff of your position is positive. (4 points)

Problem 2 (Binomial Model – 16 Points)

Consider a 2-period binomial tree with the following parameters: $S = 100$, $U = 1.10$, $D = 0.90$, $T = 2$ and $K = 100$. Suppose that the discretely compounded interest rate is 2% p.a.

a) Let p^{CoN} denote a European *binary cash-or-nothing* put option. Assume that the binary pays a flat amount of €10 if $S_T < K$, and nothing otherwise. Find the price of this option. (4 points)

b) Compute the value of the equivalent American binary cash-or-nothing put option. What is the lower price bound for this option? What is the early exercise premium of this option? (6 points)

c) Let c^{CoN} be a "call" version of the cash-or-nothing option in a). Deduce a relation between the prices of p^{CoN} and c^{CoN} *today* and use it to price the cash-or-nothing call option. (6 points)

Problem 3 (Black-Scholes Model – 18 Points)

a) A stock is currently trading at €55. The delta and the gamma of an at-the-money call on the stock can be obtained from b). If the stock price were to change to €54.75, by how much would the call price change? What is the predicted new call price? What is the approximate new value of the call delta? (6 points)

b) Suppose that you hold a portfolio of 200 shares of stock (long), 200 puts with a strike of €55 (long) and 200 calls with a strike of €60 (short). You are given the following information:

Instrument	Price	Delta	Gamma
Call with K=50	6.321	0.823	0.038
Put with K=50	0.700	-0.177	0.038
Call with K=55	3.079	0.565	0.057
Put with K=55	2.396	-0.435	0.057
Call with K=60	1.210	0.297	0.050
Put with K=60	5.465	-0.703	0.050

What is the current value of your portfolio? What are the delta and the gamma of your portfolio? (6 points)

c) Suppose that you want to make your portfolio gamma neutral. Find a least cost strategy for achieving this. (6 points)

Section 4

Problem 4 (Hedging – 10 Points)

What is meant by 'hedging'? Name one model-dependent and one model-independent hedging strategy and explain briefly how each works.

Or

Problem 5 (Swaps – 10 Points)

Explain the relationship of a swap to i) a collection of forward rate agreements (FRAs) and ii) fixed- and floating-rate bonds. What is the impact on the value of a swap if, ceteris paribus, the volatility of interest rates rises?