Examination: 20023 – Option Pricing Theory   **Winter Term 2010/2011**

Examiner: Prof. Dr. Peter Reichling

Time available: 60 minutes

Aids permitted: non-programmable pocket calculators;
English dictionaries without any markings.

The examination comprises **four** problems. All of them are to be solved. Answers must be given in **English**. Good luck!

Examination Questions (60 Points Total):

Problem 1 (Binomial Model – 19 Points)

A stock price currently quotes at €52 per share and can either rise by 8% or fall by 18% (per month) within the next **three** months. The (discretely compounded) risk-free interest rate is 5% p.a.

a) Using a binomial tree, show the possible stock price development. (3 points)

b) Determine the current value of a three-month European **put** option with strike price €48. Demonstrate the possible option price development with the help of a binomial tree. (9 points)

c) Consider the following two portfolios and prove that for an **American put** option the relation $P \leq C + K - S$ holds:

   portfolio A: one European call option plus an amount of cash equal K

   portfolio B: one American put option plus one share

(Hint: consider what will be the value of each portfolio in case the American option will be exercised and what if it will not be exercised) (7 points)

Problem 2 (Black-Scholes Model, Exotic Options – 22 Points)

Stock X shows a current price of $35 per share and a volatility of 22%. The (continuously compounded) risk-free interest rate equals 4% p.a.

a) Within the framework of the Black-Scholes model, determine the price of six-month European call and put options on the above-mentioned stock with exercise price $30. (7 points)

b) Compute the value of a European **asset-or-nothing call** option on stock X with exercise price $30 and maturity six months. In addition, find the value of a six-month **cash-or-nothing call option** on stock X with exercise price $30 and a payoff equal to the exercise price. (4 points)

c) Suppose that a European call option that expires in six months and has an exercise price of $30 can be purchased at the market for $3. With the help of an arbitrage table,

demonstrate how to execute an arbitrage strategy involving exotic options from b) and receive profit today. (6 points)

d) Explain the application of the Black-Scholes partial differential equation in hedging. (5 points)

Problem 3 (Investment Certificates – 11 Points)

a) Describe the main properties of airbag certificates. Sketch a payoff diagram of an airbag certificate. (5 points)

b) Show how an airbag certificate can be valued with the help of financial engineering. (6 points)

Problem 4 (Swaps – 8 Points)

Consider a one-year interest rate swap with semiannual payments on a notional principal of $10 million. The term structure of 180-and 360-days EURIBOR is given as follows:

Days	Rate
180	5.0%
360	5.8%

Ninety days later, at T=90, the term structure is as follows:

Days	Rate
90	4.8%
270	5.1%

The fixed rate on the swap equals 5.7% p.a. Determine the market value of the swap 90 days later (at T=90) from the perspective of the party paying the floating rate and receiving the fixed rate. All interest rates are expressed in annual terms.