Examination: 20023 – Option Pricing Theory **Summer Term 2009**
Examiner: Prof. Dr. Peter Reichling
Time available: 60 minutes

Aids permitted: non-programmable pocket calculators;
English dictionaries without any markings.

The examination is comprised of **three** problems. All of them are to be solved. Answers must be given in **English**. **Good luck!**

Examination Questions (60 Points Total):

Problem 1 (Binomial Model – 20 Points)

The price of stock X currently quotes at \$40 per share and can either rise by 10% or fall by 15% (per month) within the next three months. The (discretely compounded) risk-free interest rate is 3.5% p.a.

a) Using a binomial tree, show the possible stock price development. (3 points)

b) Determine the current value of a European **call** option with strike price \$36 and maturity three months. Demonstrate the possible option price development with the help of a binomial tree. (8 points)

c) Determine the current value of a European **put** option on stock X with the same strike and maturity as the option in sub-question b). (3 points)

d) Suppose that another stock, stock Y, currently quotes at \$25 per share. A European **put** option on this stock with strike price \$28 and maturity six months can be purchased at the market for \$1. What investment strategy would you choose to receive a profit at the put option's maturity? With the help of an arbitrage table, show the result of this strategy. (6 points)

Problem 2 (Black-Scholes Model – 20 Points)

A stock shows a current price of \$50 per share and a volatility of 22%. The (discretely compounded) risk-free interest rate equals 3.562% p.a.

a) Within the framework of the Black-Scholes model determine the price of European put and call options with exercise price \$45 and maturity four months. (9 points)

b) Check the validity of the put-call parity for the options in sub-question a). (2 points)

c) What does 'risk-neutral valuation' mean in the framework of the Black-Scholes model? Does it mean that all investors are assumed to be risk-neutral? (4 points)

d) How can one interpret $N(d_2)$ from the Black-Scholes formula? (2 points)

e) Can the Black-Scholes model be used for pricing American options? Why or why not? Explain briefly. (3 points)

Problem 3 (Trading Strategies – 20 Points)

A put option on a stock shows a strike price of \$50 and is selling for \$2. Another put option on the same stock has a strike price of \$60 and is selling for \$5. Both options have the same expiration date.

a) Sketch the profit-and-loss profile of a **bull** spread. (4 points)

b) Construct a table showing the resulting payoff from the bull spread. What kind of investors would follow this strategy? (5 points)

c) Determine the payoff and the profit from the bull spread strategy if the price of the underlying at expiration is: i) \$48 ii) \$55 iii) \$64 (3 points)

d) Determine the maximum profit and the maximum loss of the bull spread strategy as well as its break-even stock price at expiration. (4 points)

e) Can a bull spread strategy be constructed with the help of call options? If so, sketch the respective profit-and-loss profile. (4 points)

Distribution Function for the Standard Normal Distribution for Non-Negative Arguments

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767