Examination: 1928 - Derivatives **Winter Term 2007/2008**

Examiner: Prof. Dr. Peter Reichling

Time available: 60 minutes

Aids permitted: non-programmable pocket calculators; English dictionaries without any markings.

The examination is comprised of **three** problems. All of them are to be solved. **Good luck!**

Examination Questions (60 Points Total):

Problem 1 (Binomial Model – 21 Points)

A stock price quotes currently at \$50 per share and can either rise by 8% or fall by 18% (per period) within the next two periods. The (discretely compounded) risk-free interest rate is 5% p.a.

a) Using a binomial tree, show the possible stock price development. (2 points)

b) Determine the current value of an American put option with a strike price of \$45 and a maturity of two months. Demonstrate the possible option price development with the help of a binomial tree. (7 points)

c) Assume that the put option you have considered in part b) is of a European type and has a maturity of one month. Calculate the theoretical price of this option. (4 points)

d) Suppose that a European put option with the strike price of \$45 and a maturity of one month can be purchased at the market for \$3. With the help of an arbitrage table, demonstrate how to execute an arbitrage opportunity in order to receive profit today. (8 points)

Problem 2 (Black-Scholes Model – 30 Points)

A stock has a current price of \$70 per share and a volatility of 25%. The (discretely compounded) risk-free interest rate equals 3.562% p.a.

a) Within the framework of the Black-Scholes model determine the price, the delta, and the gamma of a European put as well as a European call option with an exercise price of \$68 and maturity of three months. (11 points)

b) Check if the lower bound for European put and call options is violated. If so, demonstrate how to execute an arbitrage opportunity in this case. (2 points)

c) Suppose you have a short position in 100 call options and 50 put options. What would you do to protect this portfolio against a small change in the price of the underlying asset? Which position (long/short) and in how many stocks would you take? (7 points)

d) Suppose that, having the same portfolio like in part c), you anticipate a large change in the stock price. Is this portfolio still protected? If not, show what compositional adjustment should be made. Would your portfolio still be delta-neutral? If not, show what adjustment should be made. (10 points)

Problem 3 (Trading Strategies – 9 Points)

a) A call with a strike price of \$60 has a current price of \$6. A put with the same price and expiration date has a current price of \$4. Sketch a payoff profile of a long straddle. (4 points)

b) Construct a table that shows the profit from the strategy in a). For what range of stock prices would it lead to a loss? (5 points)

Distribution Function for the Standard Normal Distribution for Non-Negative Arguments

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767