Examination: 1928 – Derivatives **Summer Term 2007**

Examiner: Prof. Dr. Peter Reichling

You are welcome to use non-programmable pocket calculators as well as English-language dictionaries without any markings. This examination comprises three problems (on two pages). All of the problems are to be solved. If you are not able to compute any values which are to be used in the following parts of a problem just assume some plausible (!) ones. We will then use your assumed values in correcting your exam. **Good luck!**

Examination Questions (Total Number of Points: 60)

Problem 1 (Binomial Model, Put-Call Parity, and Arbitrage – 23 Points)

Consider a stock with a current price of 40.00€ per share. Within one year the stock price may only rise by 10% or fall by 20%. The (discretely compounded) risk-free interest rate equals 5% p.a.

(a) Show possible developments of the stock price (in €) within one year by using a binomial tree. (1 point)

(b) Determine the price of an at-the-money European

(i) call

(ii) put

option on this stock with a maturity of one year. Again use binomial trees to show possible developments of the option prices within one year. (6 points)

(c) Verify the put-call parity for this example. What distribution assumptions concerning future stock prices do we need by using the put-call parity? Justify your answer. (4 points)

(d) Explain the implications if the risk-free interest rate was 15% p.a. (instead of 5% p.a.). (3 points)

(e) Assume again that the risk-free interest rate equals 5% p.a. Assume furthermore that the put price in the market is 1.00€. Demonstrate how to execute a "free-lunch" arbitrage strategy. (9 points)

Problem 2 (Black-Scholes Model and Delta-Gamma Hedge – 29 Points)

Consider a stock with a current price of 30.00€ per share, an expected rate of return of 10% p.a. and a volatility of 20%. The (continuously compounded) risk-free interest rate equals 4% p.a.

(a) Compute the price, the delta, and the gamma of a European

(i) call

(ii) put

option on this stock with an exercise price of 28.00€ and a maturity of six month within the BLACK-SCHOLES model. (11 points)

– please turn over –

(b) Assume that you want to invest in a portfolio consisting of the stock, the call option, and the put option so that the portfolio value is almost insensitive not only to small but also to larger changes of the stock price. Which part of your money (in %) do you have to invest in every of the three assets? Interpret your result. (What would you tell your broker to do with your money?) (15 points)

(c) Assume that you have entered into a long position in the call option. Is it possible to hedge your position by a buy-and-hold strategy just using the stock and zero-coupon bonds (with a maturity of six month) so that the value of your portfolio consisting of the call option and the hedge portfolio is almost insensitive not only to small but also to larger changes of the stock price? Justify your answer. (3 points)

Problem 3 (Asset-or-Nothing Put Options – 8 Points)

(a) Express the payoff profile of a European asset-or-nothing put option on a stock with share price S_t at time t, exercise price K, and maturity T formally and graphically. (4 points)

(b) Show how this asset-or-nothing put option can be created synthetically using European standard put options and European binary put options. How many options of what kind do you have to buy/sell? (4 points)

Distribution Function of the Standard Normal Distribution for Non-negative Arguments

x	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5675	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7034	0.7088	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8289	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8980	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9686	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9773	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817