FAKULTÄT FÜR
WIRTSCHAFTSWISSENSCHAFT

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Collective Decision Making In Organizations (20623)

Summer Term 2013
Final Exam

Solve two out of the three problems below. Each problem is worth up to 30 points. The bold figures indicate the maximum points per question. If you answer more than two problems, the first two in your working sheets will be graded, so make sure to clearly cancel out any answer you don't want to be graded (no exceptions from this procedure).

The usage of programmable calculators, textbooks, lecture notes, or dictionaries is not permitted. Notes on this exercise sheet will be disregarded during the grading.

Give answers exclusively in your working sheets; please leave a margin of 3 cm. Undecipherable scribbling will not be graded. Use the terminology and the mathematical tools presented in the lecture and the tutorial; make clear how you derive your results.

Problem 1 (30 points)

Below you find a table displaying the preferences of 100 voters (groups I, II, III, and IV) in an election with four candidates (**a**, **b**, **c**, and **d**). Each voter has one vote.

	I	II	III	IV
Number of voters:	**14**	**27**	**25**	**34**
1st rank	a	b	a	d
2nd rank	b	a	d	c
3rd rank	c	d	c	b
4th rank	d	c	b	a

a) Show that no Condorcet winner exists! (**6 points**)

b) Assuming honest and pairwise voting, explain how the agenda could be set to have candidate **a** elected. (**4 points**)

c) Under simple majority, explain why one group of voters has an incentive to vote strategically, provided all others vote honestly. (**8 points**)

Now consider the following table listing the preferences of three political groups I, II, and III, who are to vote under majority rule on the bill **A**, which stands in contrast to the status quo **B**:

	I	II	III
Number of voters:	**28**	**13**	**19**
1st rank	A	B	A
2nd rank	B	A	B

Group II anticipates **A** to receive the majority of votes and contrives to introduce an amended bill **C.** Parliamentary rules require that an amendment be first voted against the original bill (1^{st} stage), and that the survivor of this vote is then paired against the status quo (2^{nd} stage).

d) Extend the above preference table by a third alternative **C,** such that the three parties' preference relations over **A** and **B** remain the same and the status quo is obtained after the second stage vote. **(12 points)**

Problem 2 (30 points)

Suppose that the assumptions of the original Condorcet jury theorem hold. Moreover, let k denote the number of jury members, q the probability that a single member decides correctly, and $Q(k, q)$ the probability that a jury of k members (each member decides correctly with q) takes a correct majority decision.

a) Explain verbally the claims of the original Condorcet jury theorem. **(5 points)**

b) Which one of the following juries should be set in charge of decision-making if the goal is to maximize the probability of making a correct decision and appointing larger juries is costless? Rank the six juries accordingly from highest to lowest probability. **(10 points)**

Jury 1 ($k = 5$, $q = 0.6$)
Jury 2 ($k = 9$, $q = 0.63$)
Jury 3 ($k = 101$, $q = 0.48$)
Jury 4 ($k = 3$, $q = 0.6$)
Jury 5 ($k = 1$, $q = 0.6$)
Jury 6 (consisting of juries 1, 4, and 5)

c) Now suppose that a decision can be made by either of the three juries A, B, or C, where $Q_A = Q_A(1, x)$, $Q_B = Q_B(3, 0.51)$, and $Q_C = Q_C(3, 0.7)$.

i. Determine the range of x values for which A dominates B when the objective is to maximize the probability of making a correct decision. For which values of x is A preferable to both juries B and C? **(12 points)**

ii. Assume that $x = 0.51$. Consider a (von Neuman-Morgenstern-)rational and risk-neutral agent who values a correct decision at €1,000 and an incorrect decision at a zero value. What is the maximum amount the agent would pay to have the decision made by jury C, rather than by A? **(3 points)**

Problem 3 (30 points)

a) What is meant by ex-ante power? (**3 points**)

b) The (absolute) Penrose power-index and the (relative) Banzhaf power-index are common power measures. Use simple voting games as example to explain how the two indices are determined. How do these power indices relate to each other? (**15 points**)

c) Consider a voting game (W1, W2, W3; q) with W1 + W2 + W3 = 1, where Wi denotes player i's voting weight, q is the relative quorum, and B = (β1, β2, β3) is the players' Banzhaf power-index profile. For each of the following cases, provide one distribution of voting weights which yields the respective power-index profile at the given quorum. **(12 points)**

i. $q = 0.51$, B = (1/3, 1/3, 1/3)
ii. $q = 0.61$, B = (0.2, 0.6, 0.2)
iii. $q = 0.75$, B = (1, 0, 0)
iv. $q = 0.66$, B = (0.5, 0.5, 0)