PD Dr. Roland Kirstein
Professorship of Business Economics

Otto-von-Guericke University
Faculty of Economics and Management

Bargaining, Arbitration, Mediation (2622)

Winter Term 2006/07, final exam

Please solve all of the following questions 1-4. The number of points (indicated in **bold** figures) assigned to each part reflects the time you have for answering it. The total is 120.

The use of pocket-calculators, textbooks, lecture-notes, or hand-written records is neither necessary nor permitted.

1.) A monopolistic firm F transforms a single input factor L into a good Y. The production function is $Y = L$; the inverse demand for Y is $p = a - bY$, with $a > 0$ and $0 < b < 1$ (where p denotes the Y-price). F is risk-neutral and rational.

a) Assume for now that the input factor is available to F without costs. Derive the optimal input level, and the product market revenues generated thereby (denoted these hereafter as R^*). **(4 credits)**

b) Now assume that the input factor L is sold by a monopoly supplier U who seeks to maximize her revenues wL, where w denotes the factor price. U may dictate w, but F may choose the amount of the factor he wishes to purchase. Derive F's optimal reaction $L(w)$ to any possible factor price $w \in [0, a]$. **(4)**

c) What is U's optimal choice of w if she anticipates the reaction of her counterpart, as derived in part b)? Compute the equilibrium values of w and L as well as the resulting profits of F (denoted as π) and of U (denoted as V). Compare the result to the one derived in a) and discuss the difference. **(4)**

d) Draw the bargaining problem in a π-V-diagram and derive graphically (in this diagram) the symmetric Nash bargaining solution. **(16)**

e) Assume that the parties agree upon setting L at the optimal level. Hence, the only issue at stake is to bargain over w. Write down the Nash product (in payoffs) and find the SNBS. Compare your result to part d). **(12)**

2.) Ali and Baba are haggling over a cake. They ask their mother to supervise their negotiation (but, for several reasons, they are reluctant to let her decide). The mother makes the following proposal: A may divide the cake into two pieces, then B may choose one piece while A retains the other one). Both A and B sign a utility value of 1 to the whole cake; they are risk-neutral and rational.

a) Derive A's payoff function $U_A(x)$, where x denotes the size of the piece A walks away with (note that this function is contingent on B's choice!). Draw his utility function in a diagram. **(15)**

b) Derive the equilibrium of the game implemented by the mother, compare the result of this procedure with fairness concepts and the the SNBS. Explain why the equilibrium is "subgame perfect". **(15)**

3.) Two parties A and B bargain over dividing a Euro. Both are risk-neutral and rational. Derive the Asymmetric Nash Bargaining Solution and show that, for any value of $0 < \alpha < 1$, A's share is increasing *ceteris paribus* in his threat point. **(10)**

4.) At $t = 0$, an investor (A) can make a "sunk" investment (i.e., it cannot be recovered). The return of this investment amounts to $\sqrt{x}$ (and is due in $t = 2$; neglect discounting). The cost of the investment is x.

a) Derive the efficient investment. **(2)**

b) Now assume that, in $t = 1$, A has to negotiate with a business partner (B) about the distribution of the project's return. Assume furthermore that the project yields no return in case the two partners do not reach an agreement (however, A still had to bear his investment costs). As the parties want to share evenly, you may apply the SNBS. Show that A has no incentive to invest efficiently. **(12)**

c) A consultant analyzes the situation in b) and convinces the partners that the inefficiency can be avoided by using an appropriate bargaining procedure: after the investment stage, B may demand a share of the return (denote his share as β), then A may decide whether to accept or to reject B's proposal. If A is unhappy with it, he can make a counter proposal, denoted as α, but then he has to bear a penalty $p > 0$ (in your analysis you may assume that p is small). If B rejects this counter proposal, the project fails, and both parties receive zero. Draw the game tree of the game, including the investment stage (but limit the tree to representative subgames). **(10)**

d) Derive the subgame perfect equilibrium path and analyze whether the consultant's idea induces A to invest efficiently. **(10)**

e) What value of c should the consultant to choose to implement an equal split? **(2)**

f) Determine the highest fee the consultant can charge for providing his expert knowledge of game theory. **(2)**

Good luck!