Remarks:

1. The exam comprises 2 problems on 1 page. You have to solve both of them.
2. Allowed aids: bilingual dictionary.
3. Your time budget is 60 minutes.
4. Good luck!

Problem 1: Externalities and Public Goods
Consider an economy with two individuals $i = A, B$, one private good X and one public good G, with $\hat{x} = x^A + x^B$ and $\hat{g} = g^A = g^B$ being the respective quantities of the goods.

a) Consider either of the two following statements regarding the technology of the economy (which are qualitatively equivalent for the task at hand):
   - (A) The initial resource endowment of the economy comprises $\overline{x}$ units of the private good. The public good is provided by the transformation function $\hat{g} = T(\hat{x})$ with $T' < 0$ and $T'' < 0$.
   - (B) The production possibility frontier of the economy has the properties:
     - $F(\hat{x}, \hat{g}) = 0$,
     - $F(\hat{x}, \hat{g})$ is twice continuously differentiable with $F_{\hat{x}} > 0$, $F_{\hat{x}\hat{x}} > 0$, $F_{\hat{g}} > 0$ and $F_{\hat{g}\hat{g}} > 0$.

   a 1) Derive the *Samuelson condition* for this economy.

   a 2) Give an intuitive explanation of your derivation strategy and your results. How do distributional concerns influence the determination of the optimal quantity of g if the provision of the public good is financed by lump-sum taxes?

b) Consider the technology statement (A). The initial resource endowment is exhaustively distributed between the two individuals. Both individuals may now contribute voluntarily part of their endowment for the transformation into the public good.

   b 1) Describe how and why the *Nash equilibrium* in this situation differs from the optimum described by the *Samuelson condition*

   b 2) Explain the qualitative properties of the *Pigouvian* solution (tax-transfer-system) to the problem in b 1).

Problem 2: Decreasing Cost Production
Consider an economy with one household and one firm. The household's objective is to maximize utility $U = U(X, L)$, where X is the single commodity of the economy and L is labor. The firm uses (the only input factor) labor to produce the commodity via the production function $X = f(L)$. The two functions have the following properties:

$$\frac{\partial U}{\partial X} > 0, \quad \frac{\partial^2 U}{\partial X^2} < 0, \quad \frac{\partial U}{\partial L} < 0, \quad \frac{\partial^2 U}{\partial L^2} < 0,$$
$$\frac{\partial f}{\partial L} \equiv f' > 0, \quad \frac{\partial^2 f}{\partial L^2} \equiv f'' > 0 \quad \text{and} \quad f(0) = 0.$$

Assume that there exists at least one unregulated general equilibrium of X and L which is called $B = (X_B, L_B)$ which has the properties:

- both markets are cleared (with $X > 0$),
- the firm makes zero profit (average cost of producing X_B is equal to P_{X_B}).

a) Explain why B cannot be *Pareto efficient.* A graphical representation of your arguments will be helpful.

b) Draw a diagram of $f(X)$, the *Pareto efficient* allocation A and the point B in the $X - L-$space. Which (technical) properties must A fulfill? Explain the graphs that define the positions of A and B.

   Remark: You may use a single diagram for a) and b).

c) What policy intervention is needed to establish point A as a decentralized equilibrium?

d) Moving from point A to point B is associated with a welfare gain. Illustrate this gain graphically in units of L.