Examination: **Statistics I** February 11, 2004

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Please note the following:

- The exam consists of 6 problems for solution; the points you can get for each problem are given in brackets next to the number of the problem. You do not have to solve the individual problems completely, partial solutions are also possible. For problems 2-6 it is not enough, however, to state the result only, but you should clearly display your approach and way to solution.
- You can reach 50 points. For passing the exam you have to achieve a total of (at least) **22 points** from all problems.
- You are allowed to use: Pocket calculators, text books, mathematical and/or statistical tables, manuscripts and notes from lectures and/or exercises.
- **Good luck!**

Problem 1. *(5 points)*
The following statements are TRUE or FALSE. So just answer TRUE or FALSE, an explanation is not necessary.

(a) If X is a standard-normally distributed random variable, $\mu \in \mathbb{R}$ and $\sigma > 0$, then

$$Y = \frac{X - \mu}{\sigma}$$

is $N(\mu, \sigma^2)$-distributed.

(b) If two events A and B are disjoint, then they are independent.

(c) Let A, B be two events with $A \subset B$ and $P(A) > 0$. Then:

$$P(B|A) = 1.$$

(d) For any random variable X with distribution function F_X and $a, b \in \mathbb{R}, a < b$ it holds

$$P(a \leq X \leq b) = F_X(b) - F_X(a) \ .$$

(e) Let $X_1, \dots, X_n$ be independent random variables with variances $\mathrm{Var}(X_1), \dots, \mathrm{Var}(X_n)$. Then

$$\mathrm{Var}\left(\frac{1}{n}\sum_{i=1}^{n} X_i\right) = \frac{1}{n}\sum_{i=1}^{n} \mathrm{Var}(X_i) \ .$$

Problem 2. *(9 points)*
An instructor in a Statistics course set a final examination and also required the students to do a data analysis project. For 10 students the scores obtained are shown in the table.

examination scores	81	62	74	78	93	69	72	83	90	84
project scores	76	71	69	76	87	62	80	75	92	79

(a) Compute the mean, the median, the quartiles and draw a box plot for the examination data.

(b) Draw a scatterplot.

(c) Determine the empirical correlation of examination scores and project scores.

(d) From the 10 examination scores grades are computed as following:

$$\text{grade} = -0.15 \times \text{score} + 16 \ .$$

Compute the mean and the standard deviation of the resulting grade data.

Problem 3. *(8 points)*
The tread life (in 1000 miles) of a particular brand of tire has a normal distribution with mean $\mu = 35$ and standard deviation $\sigma = 4$.

(a) What is the probability of a tread life between 32000 and 38000 miles?

(b) What is the minimum tread life one can guarantee with probability 0.9 ?

Assume that you have equipped your car with four new tires of the above type. Assume independent tread lifes.

(c) State the model and calculate the probability that you can drive at least 40000 miles without having to exchange any tire because of the tread?

Problem 4. *(11 points)*
Consider a bivariate random variable (X, Y), for which the probabilities $P(X = x, Y = y)$ are given in the following table.

x \ y	1	2	3
1	$\frac{1}{9}$	$\frac{1}{9}$	$\frac{1}{9}$
2	0	$\frac{2}{9}$	$\frac{1}{9}$
3	0	0	$\frac{3}{9}$

(a) Compute the probability of the event $A = \{X \leq 2\} \cap \{2 \leq Y \leq 3\}$. Compute the conditional probability $P(\{X = 1\}|A)$.

(b) Compute the marginal distributions of X and Y.

(c) Compute the covariance and correlation of X and Y. Are X and Y independent?

(d) Compute $\text{Var}(X + Y)$.

Problem 5. *(8 points)*
Consumer complaints about a certain kitchen utensil are caused either by mechanical shortcomes or by electrical shortcomes or by appearance. Assume that

- 10% of all complaints happen during the guarantee period and are caused by electrical shortcomes,
- 10% of all complaints happen during the guarantee period and are caused by appearance,
- 50% of all complaints are caused by mechanical shortcomes. 40% of the complaints caused by mechanical shortcomes happen during the guarantee period.
- 40% of all complaints are caused by electrical shortcomes.

(a) Prove that the probability that a complaint happens during the guarantee period is 0.4.

(b) Given that a complaint happens after the guarantee period, what is the probability that it is caused by electrical shortcomes?

(c) What is the probability that a complaint is caused by appearance?

Problem 6. *(9 points)*
A fair coin is thrown n times. Let X denote the total number of 'heads' that occured.

(a) What is the distribution of X ? What is the expectation μ of X ?

Consider the event B: "X deviates more than 20% from μ".

(b) Compute $P(B)$ for $n = 10$.

(c) Compute $P(B)$ (approximately) for $n = 100$.

(d) If $n = 100$, what can be said about $P(B)$ according to Chebyshev's rule?