

Original

Professor Dr. B. Heiligers

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**Statistical Analysis I**  
**Examination**

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Please note the following

- The exams consists of 8 (equally weighted) problems for solution; for each problem you can get at most 10 points. You do not have to solve the individual problems completely, partial solutions are also possible. It is not enough, however, to give simply the result, but you should clearly display your approach and way to solution.
- For passing the exams you have to fulfill **all** of the following three requirements. You should achieve
  - a total of (at least) **25 points** from all problems, **among those**
  - (at least) **10 points from problems 1 – 4**, and
  - (at least) **10 points from problems 5 – 8**.
- You are allowed to use: Pocket calculators, text books, mathematical and/or statistical tables, manuscripts and notes from the lectures and/or exercises.

Good luck !

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**Problem 1** (10 pts)

The following stem-leaf diagram gives the salaries received by 30 students in the first year after graduation (stems represent thousands, leaves hundreds of dollars):

35		1 1 5 7
36		4 4 5 7 8 9
37		0 1 2 5 5 8
38		0 1 4 5 8
39		4 6 8
40		0 7
41		0 5
42		
43		0 1

- Calculate the median, the range, the lower and upper quartiles, and the interquartile range of the data.
- Construct a corresponding frequency histogram with five classes.

**Problem 2** (10 pts)

After an especially harsh winter, there were many potholes throughout a city's streets. The following table gives the results from inspecting 100 streets.

$m_k$	number of potholes	1	2	3	4	5
$f_k$	number of streets with this many potholes	7	24	37	19	13

- Calculate the mean and the variance of the variable "number of potholes per street".
- Reconstruction causes \$100 fixed costs per street, and \$50 for each potholes. Find the mean and the variance of the variable "total costs per street".
- The number of potholes found by examining 50 streets in a suburb has mean 2.60 and variance 1.10. Compute the overall mean and the overall variance of "number of potholes per street" for the combined sample of 150 streets.

**Problem 3** (10 pts)

Users and nonusers of legal services were asked to respond to the proposition that "prices for typical services should be displayed". The table gives the obtained results.

	agree	no opinion	disagree
Users	71	11	20
Nonusers	237	31	30

- Calculate all corresponding marginal and all conditional frequencies. Among the users, what is the percentage people agreeing to the proposition? Among the disagreeing people, what is the percentage of nonusers?
- Compute Cramer's  $V$  from the contingency table and interpret your result.

**Problem 4** (10 pts)

A fast-food chain investigated the influence of advertising expenditure on sales. For seven different regions, the table gives the relative changes, compared to the previous year, in advertising expenditure,  $x_i$ , and in sales,  $y_i$ , (both in percent); the additional columns are given for convenience.

region	$x_i$	$y_i$	$x_i^2$	$y_i^2$	$x_i \cdot y_i$
1	4	7.2	16	51.84	28.8
2	14	10.3	196	106.09	144.2
3	10	9.1	100	82.81	91.0
4	9	10.2	81	104.04	91.8
5	8	6.1	64	16.81	32.8
6	6	7.6	36	57.76	45.6
7	1	5.5	1	12.25	3.5
total	52	56.0	494	470.00	455.7

- Find the correlation coefficient between "change in advertising expenditure" and "change in sales".
- Fit a regression line to the data points, taking "change in advertising expenditure" as the regression, and "change in sales" as the response variable. Calculate the coefficient of determination.
- Predict "change in sales" when the "change in advertising expenditure" is 7 (%).

**Problem 5** (10 pts)

A new machinery must be installed before it becomes operational. The accompanying table gives a manager's probability assessment for the number of days  $X$  required for installation.

$x$	3	4	5	6	7
$f(x) = P(X = x)$	.08	.24	.41	.20	.07

- (a) Find the cumulative probability function of  $X$ .
- (b) Find the respective probabilities of the two events  
 $A$ : "Installation will take more than 4 days",  
 $B$ : "Installation will take at most 6 days".  
 What are the probabilities of the complements of  $A$  and  $B$ , respectively?
- (c) Find the probabilities of the union of  $A$  and  $B$ , and of intersection of  $A$  and  $B$ .
- (d) Are  $A$  and  $B$  mutually exclusive? Are they exhaustive?

**Problem 6** (10 pts)

Based on an investigation on a large campus, it is known that 40% of the students drink frequently in campus bars. Of those students that frequently drink, 30% have grade-point B or better, while of those students that do not drink, 50% have a B or better.

- (a) What is the probability that a student frequently drinks and has a B or better?
- (b) What is the probability that a student has a B or better?
- (c) What is the probability that a student drinks, given he has a B or better?
- (d) Are the two events "frequently drinking" and "grade-point B or better" independent?

**Problem 7** (10 pts)

From experience, a car salesman knows the following probabilities for the number of cars  $X$  that he will sell in the next week.

$x$	0	1	2	3	4	5
$f(x) = P(X = x)$	.10	.20	.35	.16	.12	.07

- (a) Find the expected value  $\mu$  and the standard deviation  $\sigma$  of  $X$ .
- (b) Find the probability that  $X$  falls in the interval  $[\mu - 2\sigma, \mu + 2\sigma]$ .
- (c) Assume that there are 10 car salesmen in the region, each of which selling cars with the above probabilities. Their sales are independent. Give the expected value and the standard deviation of the total number of sales (for all salesmen) per week.

**Problem 8** (10 pts)

It is known that 25% of all economists support a particular political party. In a group of 8 independently voting economists,

- (a) what is the probability that exactly three economists support that party?
- (b) what are the mean and the standard deviation of the total number of "supporters" (in that group)?