20024: **Microeconomic Analysis**

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Relax and take a deep breath • You are allowed 2 hours for your work • For full grade, you must solve all questions • All questions are of equal value. Their various parts, though, are not of equal weight • In your answers, you must justify your claims • The use of a calculator is permitted.

Part I *(30 Points)*

(1) Which axioms guarantee that preferences over simple lotteries can be represented by a von-Neumann-Morgenstern utility function? Give a short formal definition of the axioms!

(2) Consider the following zero-sum game:

$1 \setminus 2$	A	B	C
A	$5, -5$	$-1, 1$	$8, -8$
B	$-3, 3$	$3, -3$	$-6, 6$
C	$8, -8$	$0, 0$	$-1, 1$

(a) Find the mixed-strategy minimax equilibrium of the game!

(b) Is this game symmetric? Give a precise definition of a symmetric game!

(c) Is this game fair? Give a precise definition of a fair game!

(d) Which side-payment is necessary in order to turn this game into a fair one?

Part II *(30 Points)*

(1) Consider the following simultaneous-move game:

$1 \setminus 2$	W	X	Y	Z
A	$0, 6$	$2, 5$	$0, 2$	$6, 0$
B	$0, 1$	$0, -2$	$1, 0$	$0, 1$
C	$5, 3$	$3, 3$	$0, 2$	$5, 3$
D	$6, 0$	$2, 5$	$0, 2$	$8, 6$

(a) Give a precise definition of a strictly dominated strategy!

(b) Which of the above strategies are (iteratively) strictly dominated? Find, in each case, a strictly dominating strategy!

(c) Give a precise definition a rationalizable strategy!

(d) Which of the strategies of the game above are rationalizable? Why?

(e) Find all pure-strategy Nash equilibria of the game!

(2) Is it possible that a mixed strategy is strictly dominated by a pure strategy even though it assigns positive probability only to pure strategies that are not strictly dominated? If yes, give an example! If not, proof!

(3) Is it possible that a rationalizable strategy fails to be a best response, given only pure strategies of the opponents? If yes, give an example. If not, proof!

Part III

(30 Points)

The payoffs of two players are given by

$$\Pi_1(x_1, x_2) = (5 - 3\,x_1 + 2\,x_2)x_1 \text{ and } \Pi_2(x_1, x_2) = (10 - 3\,x_2 + 2\,x_1)x_2,$$

with $x_i \in [0, 10]$ for $i = 1, 2$.

(1) Proof - before solving the game - that a (simultaneous-move) Nash equilibrium exists and that this Nash equilibrium is unique.

(2) Do players have a first-mover or a second-mover advantage? Do players have a second-mover incentive? Again, give a short formal definition of these concepts!

(3) Find the (simultaneous-move) Nash-equilibrium of the game!

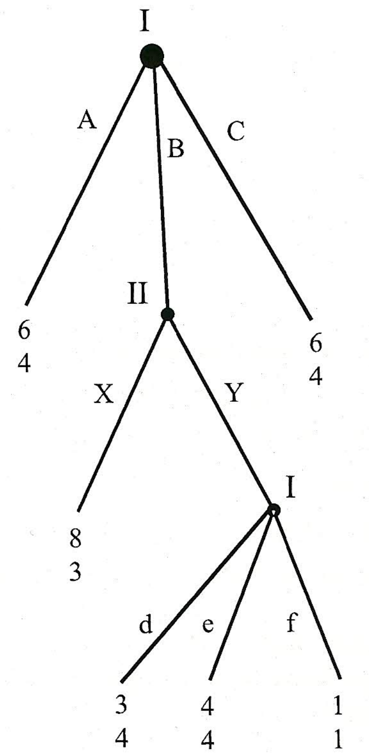
(4) Find the Stackelberg-equilibrium in which player 1 leads!

(5) Does the equilibrium in (4) Pareto-dominate the one in (3)?
(*Hint: You don't need to calculate the payoffs for this.*)

Part IV

(30 Points)

Consider the two-player extensive game given below.



(1) Specify all terminal nodes!

(2) Specify all non-terminal nodes!

(3) How many strategies does player I (II) have?

(4) How many subgames does this game have?

(5) Which strategies are payoff-equivalent?

(6) Specify all Nash-equilibria in pure strategies!

(7) What is the subgame-perfect equilibrium of the game?

(8) Identify a non-credible threat preventing a Nash equilibrium from being subgame perfect!