

Exam - 13.02.2010

Relax and take a deep breath • You are allowed 2 hours for your work • For full grade, you must solve all questions • All questions are of equal value. Their various parts, though, are not of equal weight • In your answers, you must justify your claims • The use of a calculator is permitted.

Part I

(30 Points)

- (1) Consider the following zero-sum game:

$1 \setminus 2$	A	B	C
A	$5, -5$	$-1, 1$	$9, -9$
B	$-3, 3$	$3, -3$	$-6, 6$
C	$8, -8$	$0, 0$	$-1, 1$

- (a) Find the minimax equilibrium of the following game:
 - (b) Is this game symmetric? Give a precise definition of a symmetric game!
 - (c) Is this game fair? Give a precise definition of a fair game!
 - (d) Which side-payment is necessary in order to turn this game into a fair one?
 - (e) Manipulate the payoffs, so that it becomes a symmetric game!
- (2) Proof that $m_i = -M_j$ in any two-player zero-sum game ($i, j = 1, 2$ and $i \neq j$).

Part II

(30 Points)

- (1) Consider the following simultaneous-move game:

$1 \setminus 2$	W	X	Y	Z
A	$0, 7$	$2, 5$	$7, 0$	$0, 1$
B	$5, 2$	$3, 3$	$5, 2$	$0, 1$
C	$7, 0$	$2, 5$	$0, 7$	$0, 1$
D	$0, 0$	$0, -2$	$0, 0$	$1, -1$

- (a) Give a precise definition of a strictly dominated strategy!
 - (b) Show that the strategies D and Z are (iteratively) strictly dominated! Find, in each case, a strictly dominating strategy!
 - (c) Give a precise definition a rationalizable strategy!
 - (d) Which of the strategies of the game above are rationalizable? Why?
 - (e) Find all pure-strategy Nash equilibria of the game!
- (2) Is it possible that a mixed strategy is strictly dominated by a pure strategy even though it assigns positive probability only to pure strategies that are not strictly dominated? If yes, give an example! If not, proof!
- (3) Is it possible that a rationalizable strategy fails to be a best response, given only pure strategies of the opponents? If yes, give an example. If not, proof!

Part III

(30 Points)

- (1) In the *private provision of a public good*, two players (1,2) decide whether to spend some of their private good x_i in order to supply a public G . Player i 's supply of the public good is g_i , with $g_1 + g_2 = G$. Prices are normalized to 1 and player i 's initial endowment is $\omega_i > 0$. Both players have preferences over their private good (x_i for player i) and G : $u_1(x_1, G) = x_1 + \sqrt{G}$, $u_2(x_2, G) = x_2 + \beta \sqrt{G}$, with $\beta > 0$.
 - (a) Is this a game of plain substitutes or plain complements? Is it a game of strategic substitutes or strategic complements?
 - (b) Find the best response function for both players!
 - (c) Graph the two players' best response functions for each of the following three cases: $\beta \in (0, 1)$, $\beta = 1$ and $\beta > 1$!
 - (d) Find the pure strategy Nash-equilibrium for $\beta = 2$!
- (2) Give an example for a game of strategic complements and strategic substitutes. Graph the best response functions and the iso-payoff-curves for two players for this case. Do the same (example and graph) for a game of plain substitutes and plain complements!

Part IV

(30 Points)

- (1) Specify all terminal nodes!
- (2) Specify all non-terminal nodes!
- (3) How many strategies does player I (II) have?
- (4) Which strategies are payoff-equivalent?
- (5) Specify all Nash-equilibria in pure strategies!
- (6) What is the subgame-perfect equilibrium of the game?

