Examination: 5027
Economics III
Introduction to Econometrics
Winter Semester 2007 / 2008
Dr. John E. Brennan

You are allowed to use a non-programmable calculator (in accordance with the instructions given by the examination office) and a translating dictionary from your native language to English (without any notes written into it). **All** of the **twelve** (12) examination questions must be answered (the estimated time to spend on each question is given). This examination consists of **four** (4) pages and must be completed within 120 minutes.

Answers that are NOT presented in a NEAT and ORDERLY manner, easy to read, will NOT be graded. All calculations should be rounded to four places following the decimal point (e.g., 15.6429)

Question 1 (10 Minutes)

Two of the major problems in OLS regression are hetroscedasticity and autocorrelation.

a. When autocorrelation is present are the estimated coefficients in the vector **c** biased?

b. What are the consequences of hetroscedasticy for the OLS estimate vector **c**?

Question 2 (10 Minutes)

Given a sample of 32 yearly observations (y_t, x_{t2}), the following matrices were calculated:

$$(\mathbf{X'}\,\mathbf{X})^{-1} = \begin{bmatrix} 1.606345267 & -0.01688205 \\ -0.01688205 & 0.00018094 \end{bmatrix} \qquad \mathbf{X'}\,\mathbf{y} = \begin{bmatrix} 2995.62 \\ 283964.668 \end{bmatrix}$$

Note: the following values were calculated using the correct c_1 and c_2 estimates.

$$\Sigma_t\,(y_t - c_1 - c_2\,x_{t2})^2 = 204.804524 \text{ and } s^2(Y) = 65.1762$$

$$\Sigma_s\,[(y_s - c_1 - c_2\,x_{s2})(y_{s-1} - c_1 - c_2\,x_{s-12})] = 182.265061$$

$$\Sigma_s[(y_s - c_1 - c_2\,x_{s2}) - (y_{s-1} - c_1 - c_2\,x_{s-12})]^2 = 28.299548$$

$$\Sigma_s[(y_s - c_1 - c_2\,x_{s2}) - (y_{s-1} - c_1 - c_2\,x_{s-12})] = -\,0.900113$$

$$\Sigma_t(c_1 + c_2\,x_t - \overline{y}\,)^2 = 1522.516$$

where $t = 1, 2, \ldots, T$ and $s = 2, 3, \ldots, T$.

a. Calculate the values in the (2 x 1) vector **c**.

b. Calculate the Durbin-Watson d statistic and test the H_o.

[$n = 32$, $k' = 1$: $d_U = 1.489$ and $d_L = 1.352$]

c. Calculate R^2

Question 3 (10 Minutes)

Consider the random variables Y= y and X= x with joint pdf: (X, Y) ~ f (x, y).

a. Explain in detail how the univariate random variable Y differs from the univariate conditional random variable (Y | X= x).

b. Under what conditions will these two random variables be the same?

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Question 4 (10 Minutes)

A measure of "goodness of fit" in a sample is the coefficient of determination, R^2.

a. What is the total "unexplained" variation of Y given $X=x_i$?

b. Why is R^2 thought of as a measure of the linear association between y_i and the sample estimate of the conditional expectation, $E^*(Y \mid X=x_i)$?

c. Explain in detail why R^2 should not be used when sample estimation of the BPP has been conducted.

Question 5 (10 Minutes)

You are an econometrician and you are interested in estimating the relationship between private consumption and family income. Annual data has been obtained from the responsible statistical agency from 1920 to 1990. Some of your colleagues, however, have indicated to you that your study should take into account the Second World War period (1939 – 1945).

a. Explain in detail a BLP estimation procedure that would allow the estimated level of consumption to be different before, after, and during the war years. Describe the **y** vector and the **X** matrix that would be used and highlight any estimation problems that might arise as a result of your specification.

b. Explain in detail a BLP estimation procedure that would allow the estimated marginal propensity to consume (mpc) to be different. Describe the **y** vector and the **X** matrix that would be used and highlight any estimation problems that might arise as a result of your specification.

Question 6 (10 Minutes)

Given the discrete joint bivariate probability distribution for the random variables X and Y.

Y \ X	8.2	13.4	18.4	23.4	$f_2(y)$
1.7	0.035	0.058	0.072	0.125	
3.7	0.081	0.091	0.058	0.085	
5.7	0.092	0.069	0.068	0.166	
$f_1(x)$					

Given: $E(X) = 17.0684$, $E(X^2) = 326.0474$, $E(XY) = 66.15428$

a. What is your "best" MSE prediction for the value of Y when x = 13.4?

b. What is your BLP prediction of Y knowing that x = 13.4?

c. What is your BPP prediction of Y knowing that x = 13.4?

d. Is Y mean independent of X? Explain your answer in complete detail.

Question 7 (10 Minutes)

Stochastic independence and mean independence both imply that the $C(X, Y) = 0$.

a. If X and Y are stochastically independent, explain the implication for the CEF and the BLP.

b. Explain the difference between stochastic and mean independence.

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Question 8 (10 Minutes)

A perfect linear relationship among some or all of the explanatory variables in the **X** matrix, $\lambda_1 X_1 + \lambda_2 X_2 + ... + \lambda_k X_k = 0$, for the constants λ, when not all are equal to zero, is called multicollinearity according to Ragnar Frisch (1934).

a. What are the consequences of perfect multicollinearity for the OLS estimates?

b. What is the consequence of one or more "near" exact relationships among the columns of the **X** matrix for the OLS estimates? Explain in detail.

Question 9 (10 Minutes)

The Runs Test (Geary Test) is a nonparametric test that can be used to check if the conditional random variable $(Y \mid X = x_t)$ is correlated with $(Y \mid X = x_{t-1})$. An OLS regression was run on a sample and the following 15 prediction errors, e_t, were calculated.

t	e_t	t	e_t	t	e_t
1965	+ 0.1875	1970	+ 0.2134	1975	+ 0.2187
1966	− 0.3482	1971	− 0.0421	1976	+ 0.0463
1967	− 0.0173	1972	− 0.6234	1977	− 0.2419
1968	+ 0.4713	1973	+ 0.1267	1978	− 0.1234
1969	− 0.0964	1974	+ 0.1167	1979	− 0.0853

a. Compute and explain the rationale behind the Runs (Geary) Test. $n_1 + n_2 = N$
$E(\delta) = [(2\, n_1\, n_2) / N] + 1$ and $V(\delta) = \{2\, n_1\, n_2\, [(2\, n_1\, n_2) - N]\} / \{N^2\, (N - 1)\}$

b. Can the null hypothesis be accepted in this case? Explain your answer fully.

Question 10 (10 Minutes)

$$\underset{(n \times n)}{\text{Var-Cov}(Y \mid \mathbf{X})} = \begin{array}{c|cccccc} & 1 & 2 & \dots & t & \dots & n \\ \hline 1 & \mathbf{V(Y|X_{1j}=x_{1j})} & C(Y|X_{1j}=x_{1j}, Y|X_{2j}=x_{2j}) & & C(Y|X_{1j}=x_{1j}, Y|X_{tj}=x_{tj}) & & C(Y|X_{1j}=x_{1j}, Y|X_{nj}=x_{nj}) \\ 2 & C(Y|X_{2j}=x_{2j}, Y|X_{1j}=x_{1j}) & \mathbf{V(Y|X_{2j}=x_{2j})} & & C(Y|X_{2j}=x_{2j}, Y|X_{tj}=x_{tj}) & & C(Y|X_{2j}=x_{2j}, Y|X_{nj}=x_{nj}) \\ t & C(Y|X_{tj}=x_{tj}, Y|X_{1j}=x_{1j}) & C(Y|X_{tj}=x_{tj}, Y|X_{2j}=x_{2j}) & & \mathbf{V(Y|X_{tj}=x_{tj})} & & C(Y|X_{tj}=x_{tj}, Y|X_{nj}=x_{nj}) \\ n & C(Y|X_{nj}=x_{nj}, Y|X_{1j}=x_{1j}) & C(Y|X_{nj}=x_{nj}, Y|X_{2j}=x_{2j} & & C(Y|X_{nj}=x_{nj}, Y|X_{tj}=x_{tj}) & & \mathbf{V(Y|X_{nj}=x_{nj})} \end{array}$$

a. Explain the conditions that are necessary for the covariances in this matrix to be exchanged for correlation coefficients.

b. Completely define the First-Order Autoregressive Process, AR(1). What is the value of Corr $[(Y \mid X_{tj} = x_{tj}), (Y \mid X_{t-5j} = x_{t-5j})]$?

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Question 11 (10 Minutes)

The Linear Probability Model states that in the population:

$E(Y \mid Z = z_i, W = w_i) = \beta_1 + \beta_2 z_i + \beta_3 w_i = p_i$ and $V(Y \mid Z = z_i, W = w_i) = p_i (1 - p_i)$ where $p_i = \Pr(Y = 1 \mid Z = z_i, W = w_i)$. The dichotomous random variable Y signifies home ownership, the continuous random variable Z measures disposable income, and the dummy variable W is equal to one if the individual has an advanced degree from an institution of higher learning.

a. Critically discuss this model and any estimation problems that might be involved.

b. Is autocorrelation likely to be present?

Question 12 (10 Minutes)

Consider the random variable X ~ Rectangular (continuous uniform) $a < b$

$$f(x) = 1/(b - a) \text{ for } a \le X = x \le b, \text{ with } f(x) = 0 \text{ elsewhere.}$$

$$F(x) = (x - a)/(b - a) \text{ for } a \le x \le b\text{: } F(x) = 0 \text{ for } x < a \text{ and } F(x) = 1.0 \text{ for } b < x.$$

Calculate the probability of each of the following events for $a = 0$ and $b = 3$.

a. $A = \{0 \le X = x \le ½\}$

b. $B = \{¼ \le X \le ¾ \mid x \le 2\}$

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This is the end of the examination.

GOOD LUCK !!